





PERCEPTIONS

Changing the Way You Look At Math



Intensive Mathematics Intervention
Multiplication, Division, & Fractions

Instruction Manual

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Daily Sequence

This is designed as a 16-week intensive intervention for students entering 5th grade or higher. A basic understanding of multiplication and division facts is helpful.

- *Built-in Progress Monitoring for every lesson*
- *16 weeks equals 80 days*
- *Each lesson is designed to take 2 days, although some may be one or three days. The 32 Lessons equal 64 of the 80 days, allowing for days of testing and days for application, additional practice, or make-up.*

Example: Unit 1

Day 1 Level Pre-Test

Day 2 & 3 Unit Pre-Test & Lesson 1

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Day 14 Unit 1 Post-Test

- Purpose, Goal, Methodology, and Philosophy
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Lesson 32: Solving for Unknowns in Fractions

Lesson 33: Solving for Unknowns (**Optional**)

Purpose, Goal, and Philosophy

Perceptions - A 16 Week Intensive Intervention

by Teach 4 Mastery, Inc.

Purpose (Reason) for this Intervention

Research¹ has shown that a student's understanding of fractions and whole-number division are early predictors of the student's mathematical achievement in high school. This was found to be more important than the student's knowledge of other mathematical areas along with other social and economic factors.

The research further supported that mastery of fractions and division is important if students are to develop understanding of algebra and other aspects of high school mathematics.

The Goal of this Intervention

1. Provide a supplementary intervention covering multiplication, division, and fractions
2. To increase the conceptual understanding of fractions and division in teachers
3. To increase the conceptual understanding of fractions and division in students

Methodology and Philosophy - *Construct, Express, & Apply*

Construct: Demonstrate through both concrete and semi-concrete representations of mathematical concepts and ideas

Whenever the instructions indicate to *construct*, we recommend using the *MasterPieces™* or the *MasterFractions™* to **build** or **draw** the problem given and develop conceptual understanding.

Express: Demonstrate through artistic, written, and verbal expression, fluency and accuracy of the concept and computation of the mathematical process

Whenever the instructions indicate to **solve, show, draw, etc...** we recommend using **drawing, written,** and **verbal** communications to have students *express* what they have learned. Get creative on how the students will express to you or to others the understanding they have of a given concept and process. Ideally, a student who has mastered a concept can hear the problem verbally and express back the computation, concept, and application of the problem in a written or verbal manner.

Apply: Demonstrate the correct application of the mathematical concept and computation into real world settings

Word Problems & Mastery Challenges - Being able to apply what has been learned is the greatest sign of achievement!

¹A 20 year longitudinal study on Early Predictors of High School Mathematics Achievement by Robert S. Seigler et al.

Lesson 5: Beginning Division

Lesson Objectives:

1. Understand that division is the inverse operation of multiplication
2. Understand that the divisor and quotient are new names for factors
3. Understand that dividend is the new name for our product
4. Learn the following terms or symbols:

Inverse

Multiplication

Division

Divisor

Dividend

Quotient

Lesson Steps:

Step 1: Both Teacher and Student Watch Video

**Step 2: Explicitly Instruct Students Using Manipulatives
Construct, Express, & Apply**

Step 3: Student Proceeds to Workbook Pages for Practice

Step 4: Student Demonstrates Mastery - Teaching Back to You

Now take what we have learned about multiplication and use it in the inverse operation known as **Division**.

The term **inverse** means to reverse the position, order, or direction of what you have been doing. With that in mind look at these operations:

Multiplication: increasing one factor by another factor to determine the product

Division: given the **dividend** (product) and the **divisor** (one factor), now look to find how many times the divisor can be counted out of the dividend

In multiplication we are looking for the product, and in division we are looking for the missing factor (**quotient**). It is important to realize the connection between these two operations.

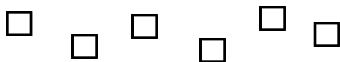
$$2 \times ? = 6 \text{ is the same as } 6 \div 2 = ?$$

Example (*Fig A*): We have 6 units, and we want to know how many groups of 2s we could count out of 6. We could make 3 equal groups of 2s. If we were to put those 3 groups together, what shape would they make? A rectangle

As a helpful word association, the words Quotient and Question both begin with the letter "Q." In division, what is the question? The Quotient!

Example

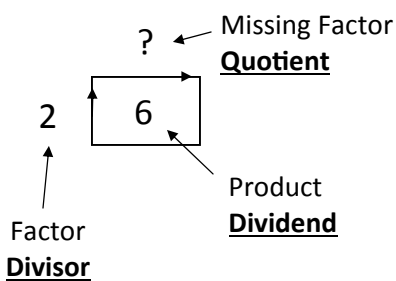
Fig. A How many 2s out of 6?



Make Equal Groups

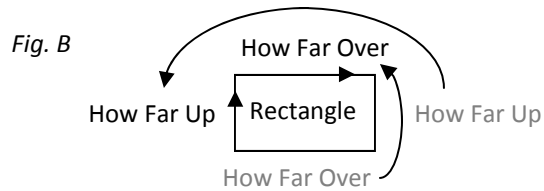


Make a Rectangle

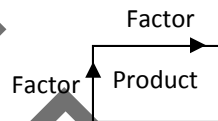


Lesson 5: Beginning Division

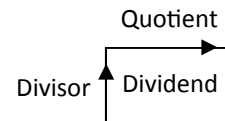
The rectangle (*Fig. B*) is what makes up the area representing the dividend. Since we are doing the inverse operation (division) of multiplication, let's move our factors from the bottom and right to the top and left.



Multiplication Terms



Division Terms



The key is to realize that they are both representing the same thing. The only difference is whether you know how much you have (Dividend) or only the factors.

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Student Workbook

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Student Self-Assessment Rubric

We at Teach4Mastery are so excited to help you understand math better, and we hope our methods and instruction will make it easier for you.

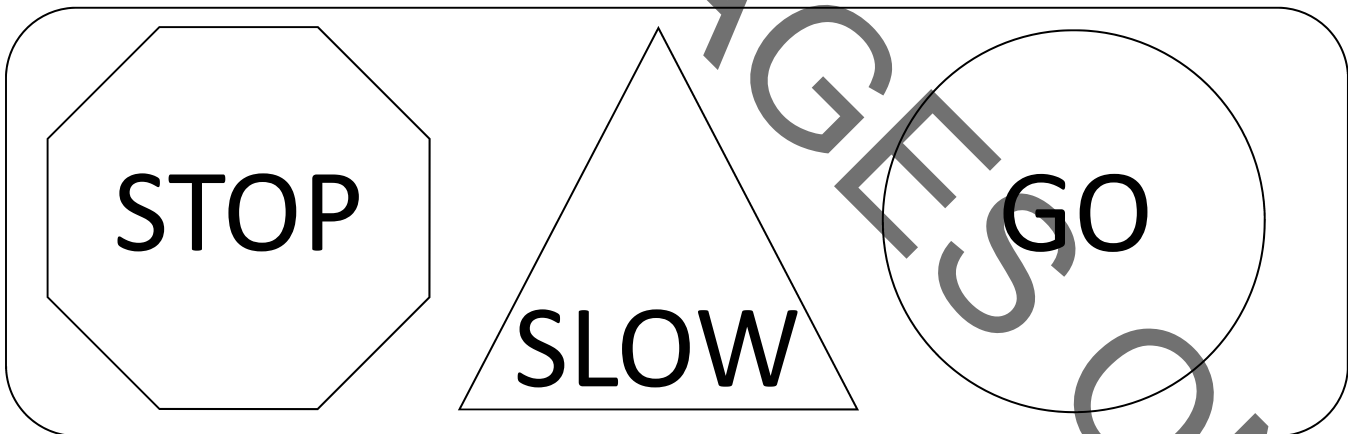
On the back side of each of your student pages, you will find the following shown below. This will assist you in letting your teacher know if you are understanding the materials or need additional help.

After you complete each page, shade in the symbol that best describes how you feel about the work you just finished.

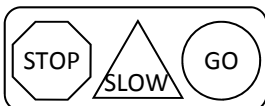
STOP means "I don't get it. I need more help in understanding the concept or completing the problems."

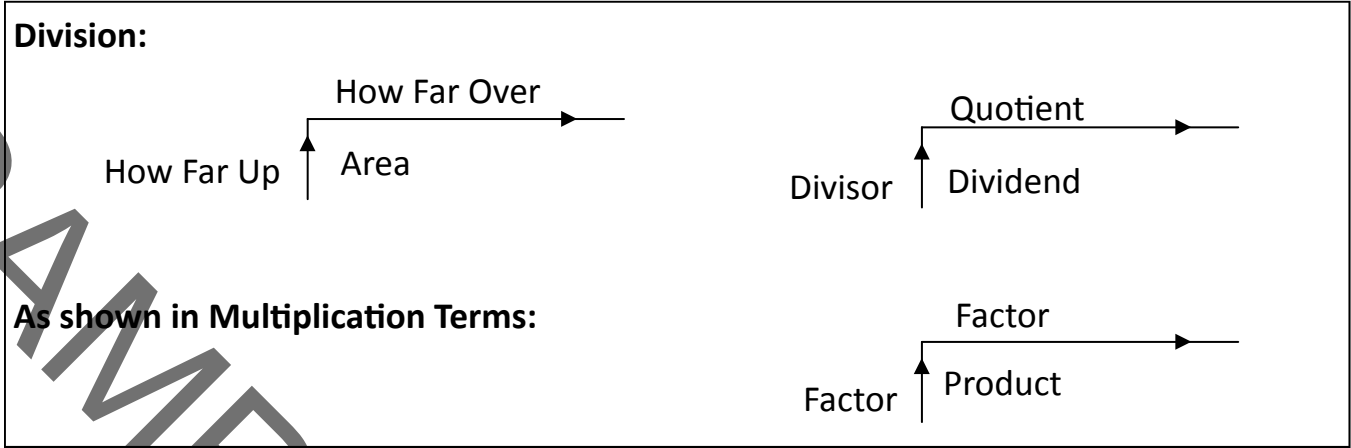
SLOW means "I'm beginning to understand, but I will need additional work to be able to complete the problems with confidence."

GO means "I got it! I understand the concept and can do the problems given with confidence."

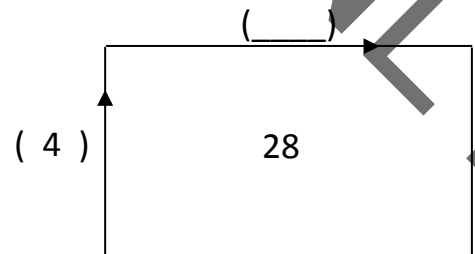
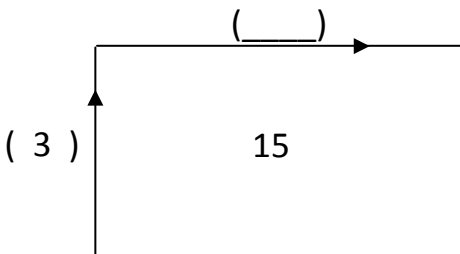
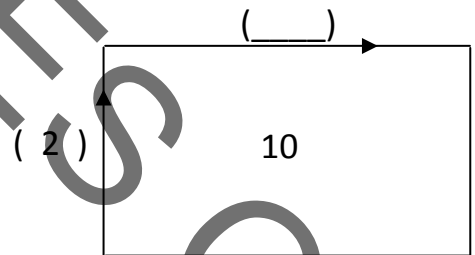
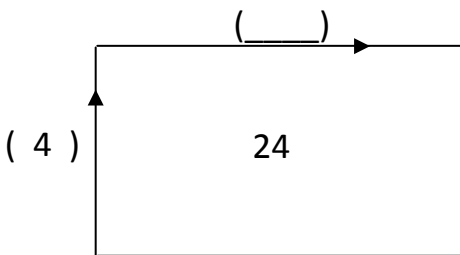
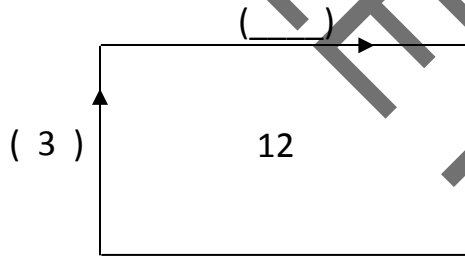


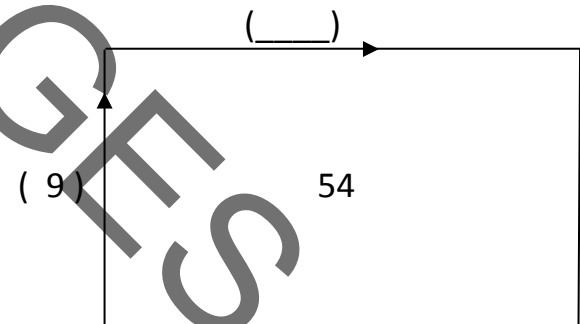
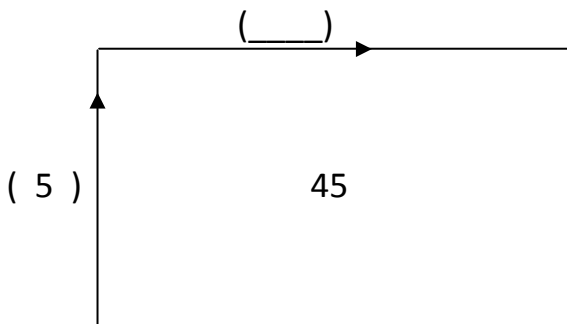
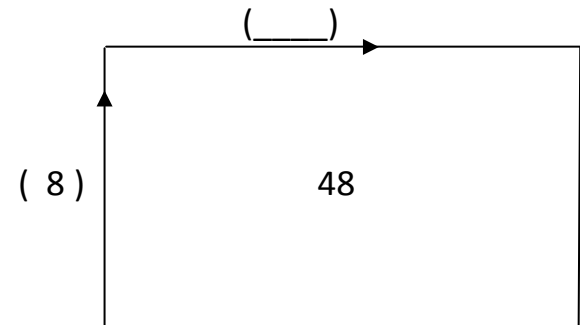
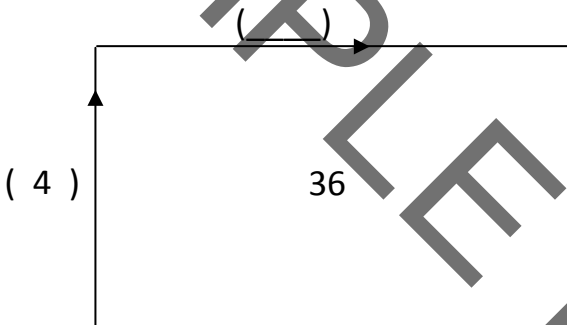
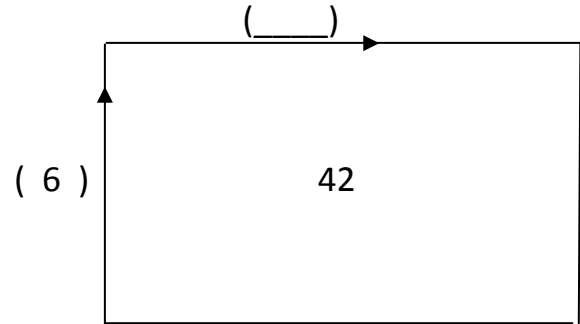
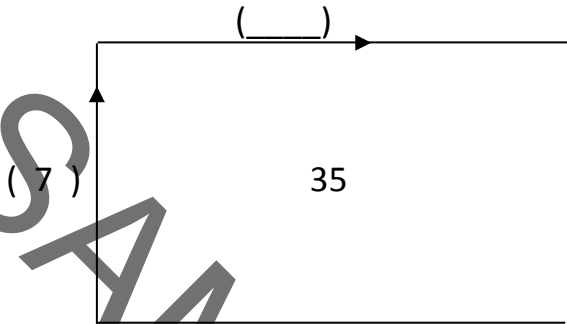
Here is where you will find your student rubric in your workbook. Ask your teacher if you have any questions.





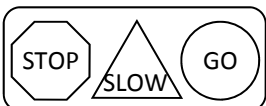
Construct, then use the rectangles to draw your lines to show your answer.



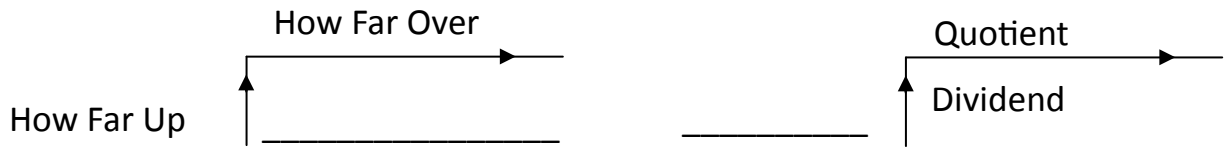


Adrienne made 36 cupcakes for Emmie's birthday. If there are 12 people at her party, how many cupcakes could each person have?

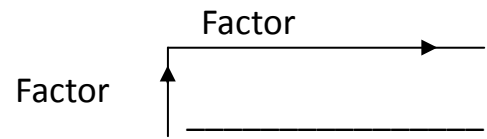
Maggie is going to Grandma and Grandpa's house for one week. Her mother packed 14 outfits. If she wore the same amount of outfits each day she was there, how many outfits could she wear each day?



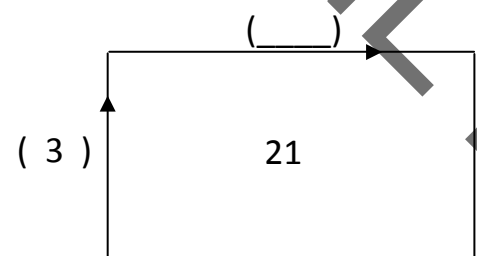
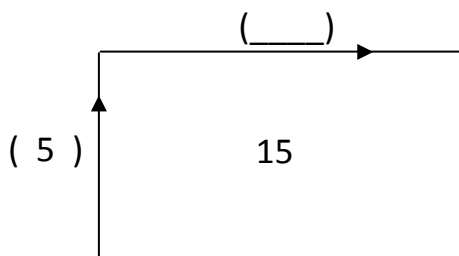
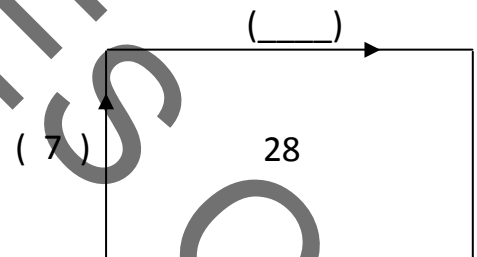
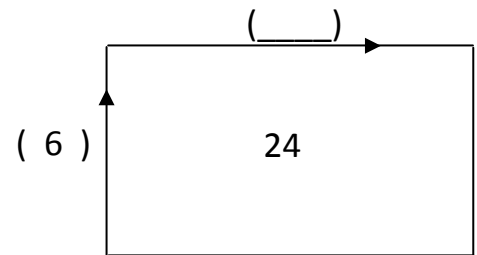
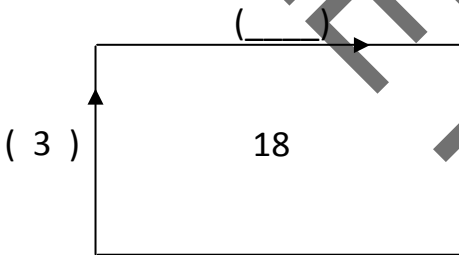
Division:

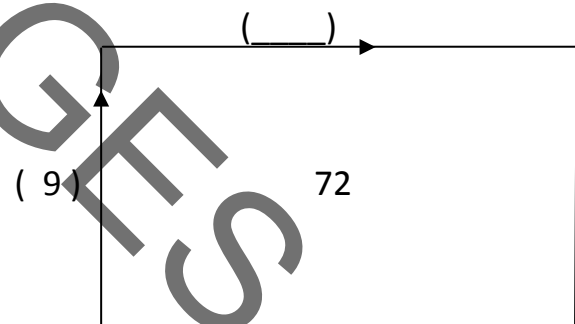
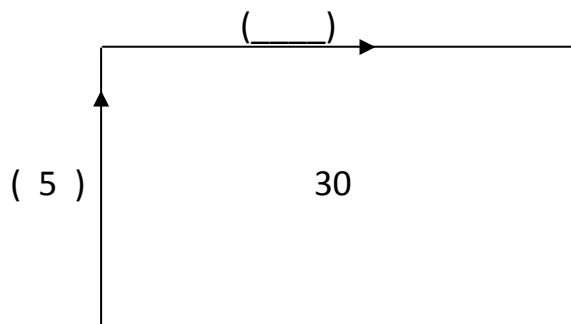
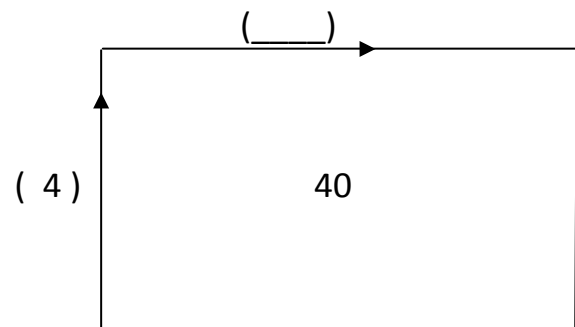
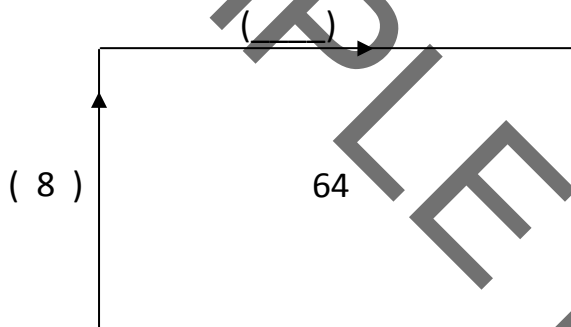
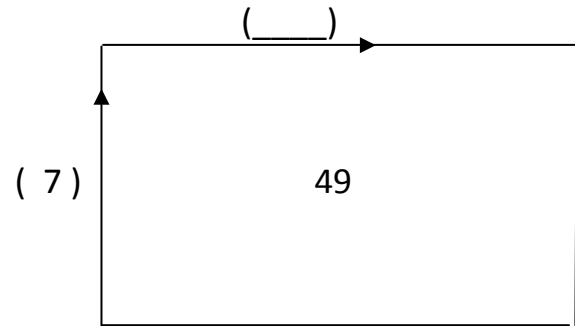
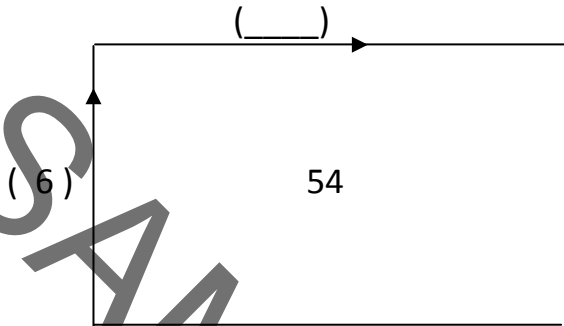


As shown in Multiplication Terms:



Construct, then use the rectangles to draw your lines to show your answer.



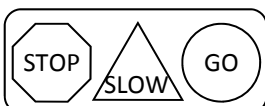


Bentley's grandmother made 16 cookies for Bentley and her friends.

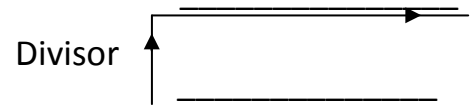
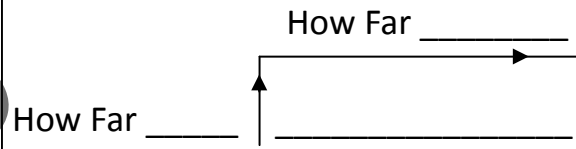
If Bentley has one friend over,
how many cookies will each person get?

If Bentley has three friends over,
how many cookies will each person get?

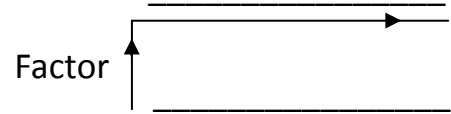
Bobby's gumball machine only had 36 gumballs left in it. Each time he turned the lever, 4 gumballs came out. How many times can he turn the lever before the machine will be empty?



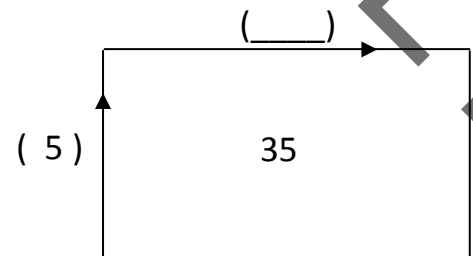
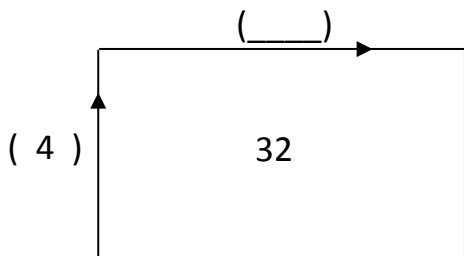
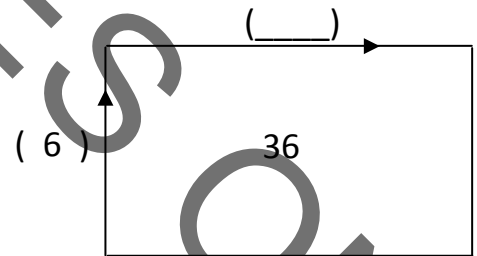
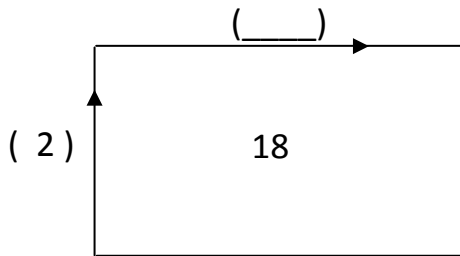
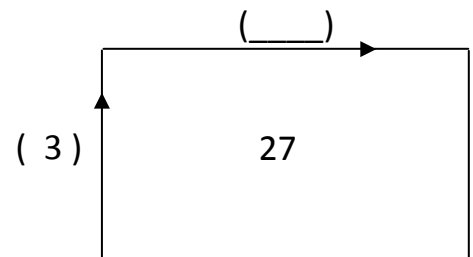
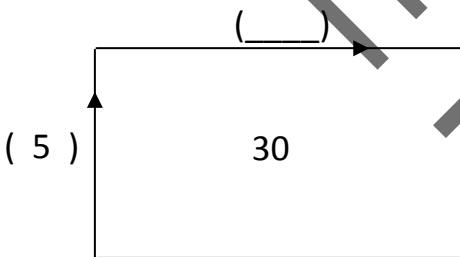
Division:

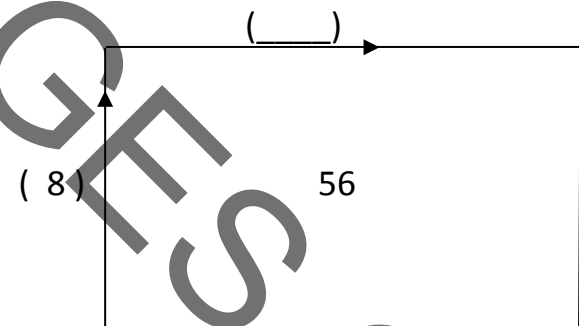
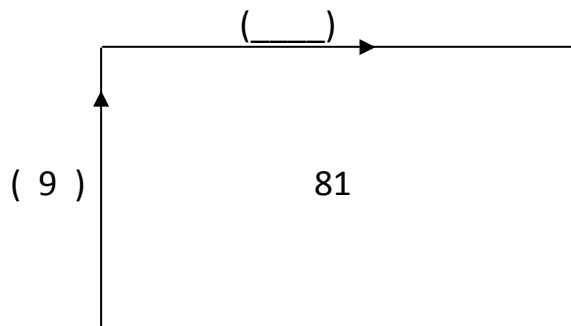
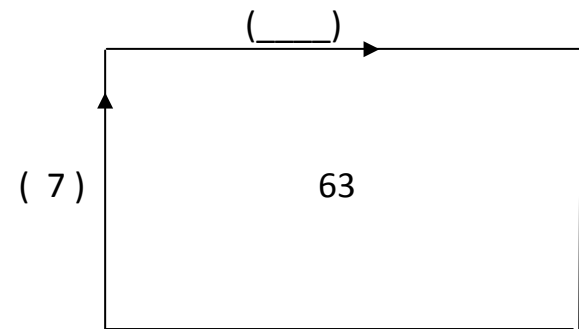
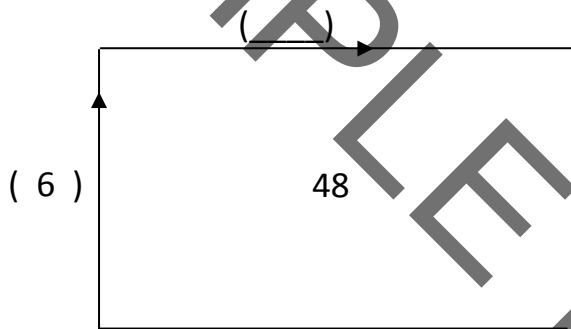
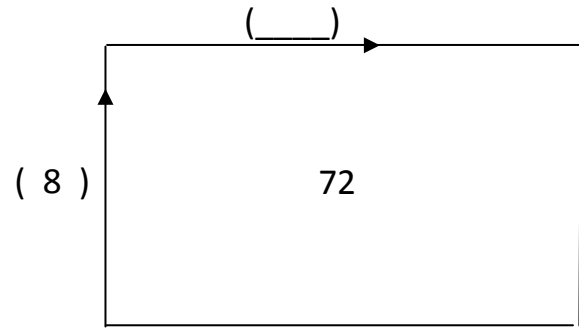
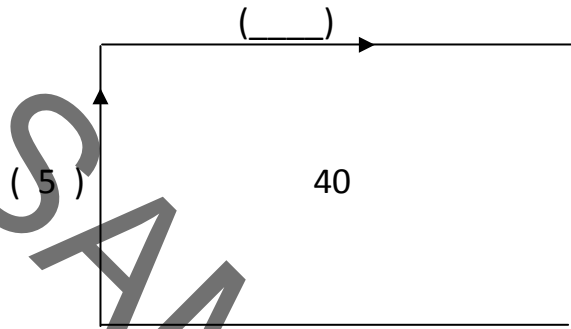


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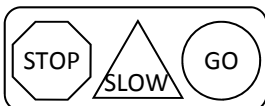
Construct, then use the rectangles to draw your lines to show your answer.





Michelle bought the family size pack of toilet paper that had 48 rolls. If her family goes through 8 rolls a week, how long will that family pack last?

Lizzy had 54 old CDs. She decided that was too many. So, she divided them up evenly among 6 friends. How many CDs did each of her friends get?



Mastery Challenge 5

Ezekiel's total budget for his birthday party was \$100. He wanted to go to the zoo with his friends. Each ticket cost \$6.00 each. Since they were going to be there all day, he would need to purchase lunch for each person. Each lunch ticket was \$4.00. How many friends could he bring if both of his parents were also coming?

Jessica processed 18 shipments in 3 hours. How many shipments did she average per hour?

If the company sent out an average of 24 shipments per hour, how many employees did they have processing shipments? (Please assume that everyone ships the same amount as Jessica per hour.)

Nick was training for a marathon. He needed to run 54 miles per week. He ran in the mornings on Monday, Wednesday, and Friday, and then ran in the afternoon on Tuesday, Thursday, and Saturday. On average, how many miles would he need to run in each day in order to meet his goal?

In addition to his running, his training coach suggested that he ride his bike 48 miles per week. If he rode his bike each day that he ran, how many miles would he need to bike each day to complete this goal?

How many miles was Nick running and biking each day?

Annette's grandmother was a professional caterer. Her grandmother gave her the recipes and the shopping lists for them, however all of her lists were for 16 people. Annette needed to serve her family of only 4 people.

What number must Annette divide by to convert her grandmother's recipes for her family? _____

2 cups = 1 pint 2 pints = 1 quart 4 quarts = 1 gallon

Her grandmother's shopping list called for 1 gallon of milk, how many quarts would Annette need to purchase? _____

If the milk only came in pints, how much would she need? _____

If you converts the pints into cups, how many cups of milk would she need? _____

Her list also called for...

Two pints of melted butter, how much butter would Annette need? _____

One dozen eggs, how many eggs would Annette need? _____

One quart of sour cream, how much sour cream would Annette need? _____

Two quarts of buttermilk, how much buttermilk would Annette need? _____

